Mixture Models with Missing data Classication of Satellite Image Time Series

Serge Iovleff, Mathieu Fauvel, Stéphane Girard, Cristian Preda, Vincent Vandewalle

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Classification of Satellite Image Time Series
QUALIMADOS: Atelier Qualité des masses de données scientifiques

S. Iovleff, Mathieu Fauvel, Stéphane Girard, Cristian Preda, Vincent Vandewalle

Laboratoire Paul Painlevé

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Sommaire

Clustering is the cluster building process

Cluster analysis

From Wikipedia, the free encyclopedia

For the supervised learning approach, see Statistical classification.

Cluster analysis or **clustering** is the task of grouping a set of objects in such a way that objects in the same group (called a **cluster**) are more similar (in some sense or another) to each other than to those in other groups (clusters). It is a main task of exploratory data mining, and a common technique for statistical data analysis, used in many fields, including machine learning, pattern recognition, image analysis, information retrieval, bioinformatics, data compression, and computer graphics.

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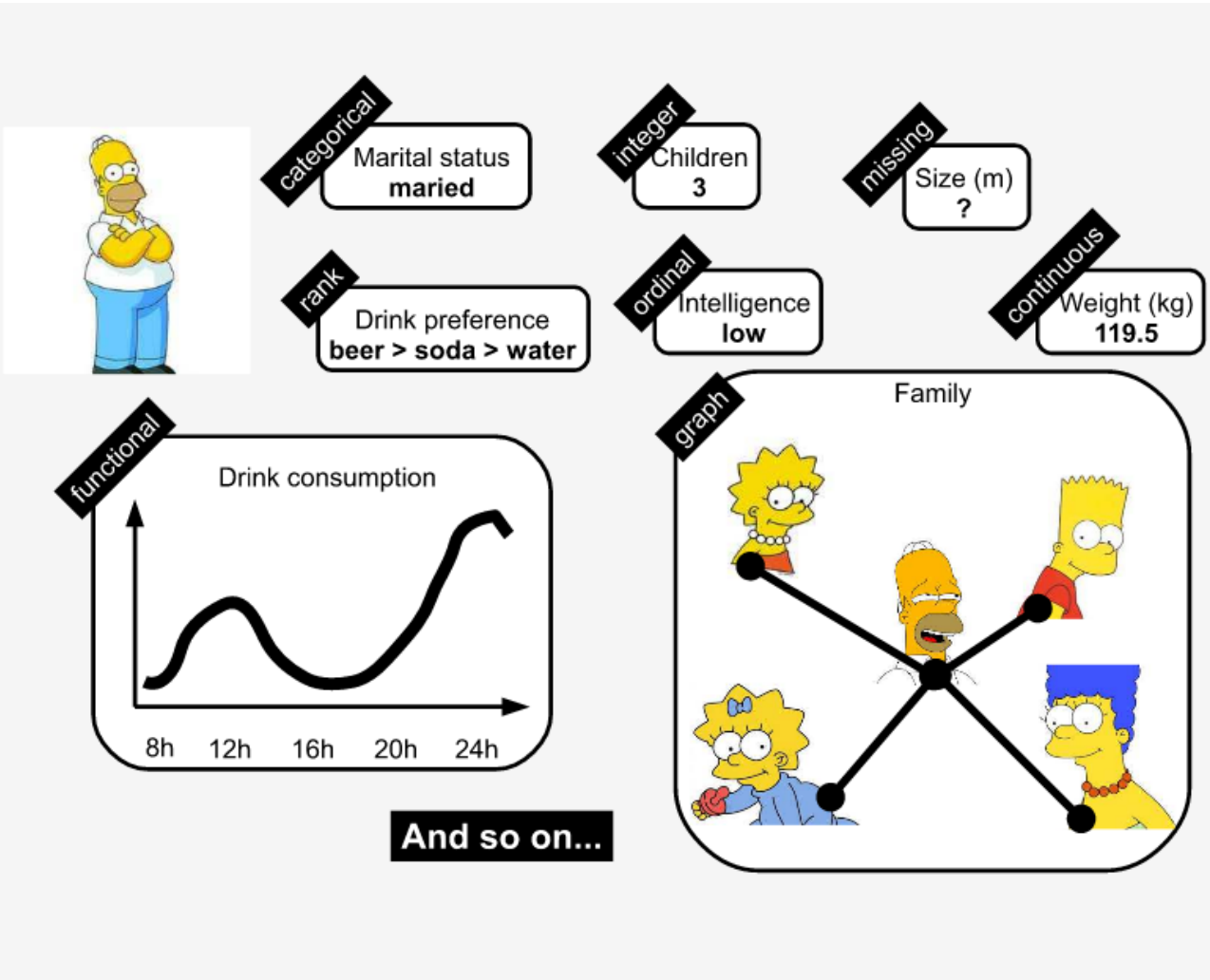
New challenges

Need to algorithms for Big-Data and Complex Data. In particular mixed features and missing values



An example

Joint works with Christophe Biernacki (head of the Inria Modal team), Vincent Vandewalle, Komi Nagbe,...

Contract for a large lingerie store: "*Clustering cash receipts of the Customers with a loyalty card*"

- 28 variables related to products,
- 6 variables related to costumers,
- 8 variables related to stores,
- $n = 2,899,030$ receipts.

Some meaningful variables with missing values.

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Some meaningful variables with missing values.

An example (Variables)

Variables liées aux clients

Num	Name_var	Type	Nbre Modal	Modalités	% Manquant	Discrétisé
1	CODE_CIVILITE	factor	4	MLE|MM|MME|MR	32.55	NON
2	TRANCHE_URBAINE	factor	5	+100000|20000 à 50000|-5000|50000 à 100000|5000 à 20000	33.37	NON
[illegible]	[illegible]	factor	2	NON|OUI	0	NON
[illegible]	[illegible]	factor	2	NON|OUI	0	NON
51	ANNEE_NAISSANCE (Âge)	numeric	106		0 37.07	NON
[illegible]	[illegible]	integer	225		0 32.55	NON

Variables liées aux magasins

Num	Name_var	Type	Nbre Modali	Modalités	% Manquant	Discrétisé
1	TYPE_MAGASIN	factor	2	FRANCHISE|SUCCURSALE	0	NON
2	SECTEUR_MAGASIN	factor	10	[illegible]		NON
3	LOCALISATION_MAGASIN	factor	4	CENTRE CIAL AVEC HYPERMARCHE|CENTRE CIAL SANS HYPERMAR	0.41	NON
4	MUSIQUE_MAGASIN	factor	2	NON|OUI	0	NON
5	magasin_ferme	factor	2	FERME|NON FERME	0	NON
6	[illegible]	factor	4	1-5ans|6-10ans|11-15ans|plus de 15ans	0	NON
[illegible]	[illegible]	factor	28		0	NON
8	SUPERFICIE_MAGASIN	integer	53		0 0	NON

An example (Variables)

Variables liées aux Produits

Num	Name_var	Type	Nbre Moda	Modalités		% Manqua	Discrétis
1	mois_ticket	factor	12	01|02|03|04|05|06|07|08|09|10|11|12		0	0
2	jours_ticket	factor	7	dimanchelieudi|lundi|mardi|mercredi|samedi|vendredi		0	0
3	TYPE_LIGNE	factor				0	0
5	MONTANT_REMISE	factor		EMISE|SANS REMISE		0	OUI
6	SOLDE	factor	2	NON|OUI		0	0
7	MT_REMISE_OP_COMM	factor	2	REMISE COMM|SANS REMISE COMM		0	OUI
		factor	2	NON|OUI		0	0
9		factor	2	NON|OUI		0	0
10	REGLEMENT_CB	factor	2	NON|OUI		0	0
11	REGLEMENT_ES	factor	2	NON|OUI		0	0
12	REGLEMENT_CH	factor	2	NON|OUI		0	0
13	REGLEMENT_AUTRES	factor	2	NON|OUI	"REGLEMENT_KK","REGLEMENT_CB","REGLEMENT_ES","REGLEMENT_CI	0	0
14	KKDO_UTILISATION	factor	2		NON|OUI	0	0
1		factor	1		NON	0	0
16		factor	2	NON|OUI		0	0
17		factor	2	NON|OUI		0	0
18	ACT[illegible]VELLE_CLIENTE	factor	2	NON|OUI		0	0
1		factor	2	NON|OUI		0	0
20		factor	2	NON|OUI		0	0
21	GROUPE_PRODUIT	factor	5		ACCESSOIRES MODE|BALNEAIRE|BAS COLLANTS|LINGERIE DE JOUR|LIN	0	0
22	COLLECTION	factor	2		[illegible]	0	0
23	COLORIS_BASE	factor	13		Blanc|Bleu|[illegible]une|Marron|Noir|Orange|Peau|Rose|Rouge|Vert|Viol	0	0
24	COORDONNE	factor	4		[illegible]NUIT|JOUR|[illegible]	19.02	0
25	STYLE_PORTER	factor	4		CONFOR[illegible]SEDUCTION|SEXY	19.74	0
26	ASPECT_COLORIS	factor	3		BICOLORE|IMPRIME|UNI	2.35	0
27	CARACT_MATIERE	factor	5		AUTRES|DENTELLE|MICROFIBRE|SATIN|TULLE BRODE	0	0
4	PRIX_UNITAIRE	numeric	4811		0	0	0

An example (Results)

	Classe1	Classe2	Classe3	Classe4	Classe5	Classe6	Classe7
Proportion	[redacted]	18.31%	[redacted]	[redacted]	14.77%	[redacted]	6.66%
Effectif	[redacted]	527499	[redacted]	[redacted]	427417	[redacted]	205443

An example (Results)

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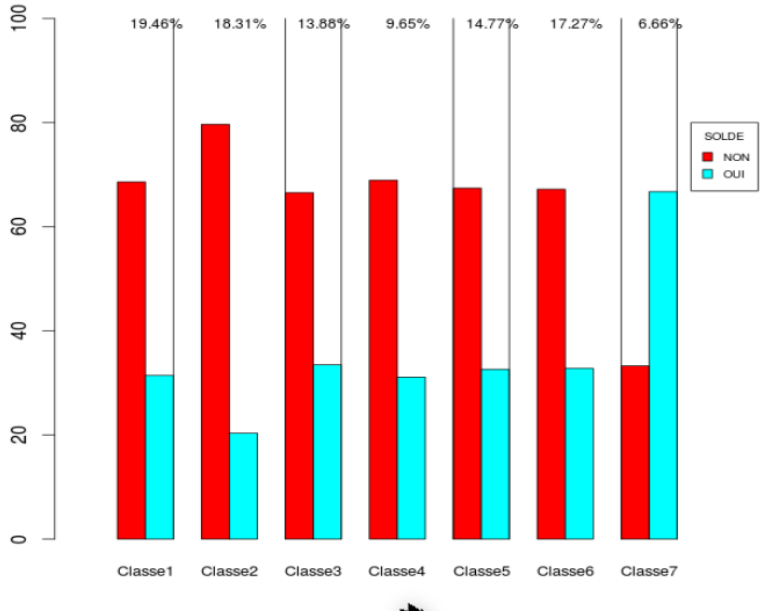
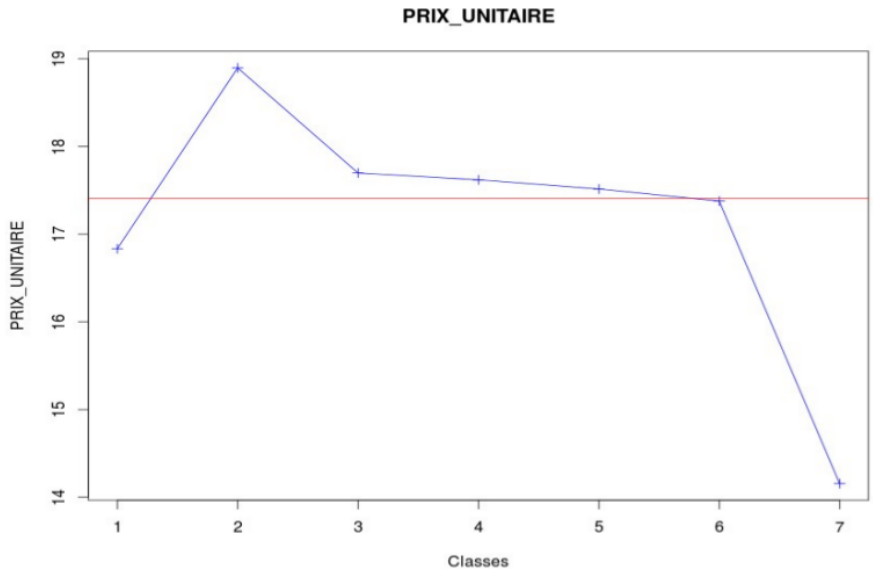
Solde



Prix Unitaire



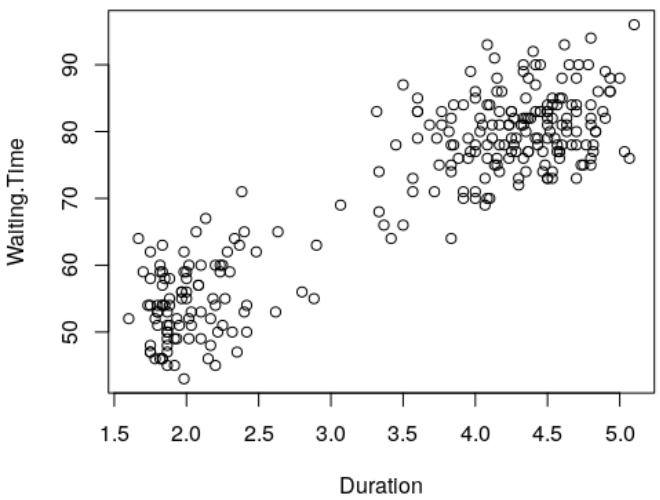
Mixture Models

Main Idea

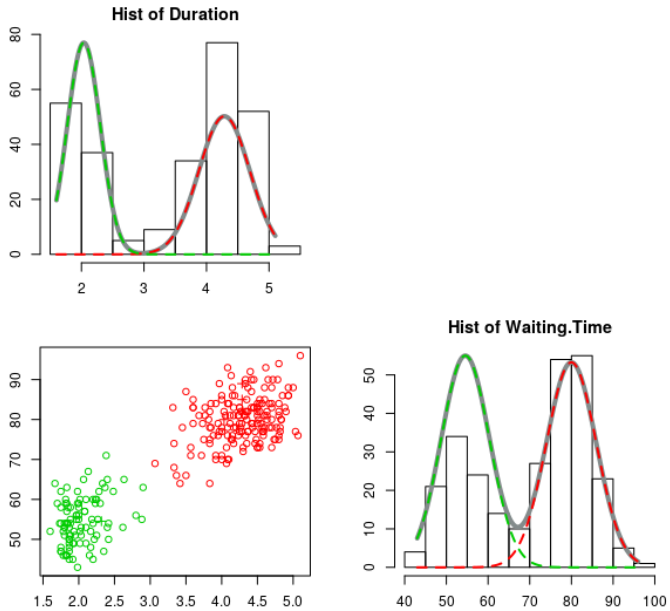
$\mathbf{x}$ in cluster $k \iff \boldsymbol{x}$ belongs to distribution P_k

$\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$

$\hat{\mathbf{z}} = (\hat{\mathbf{z}}_1, \ldots, \hat{\mathbf{z}}_n)$, $\hat{K}$ clusters



$\longrightarrow$ clustering



Model Based clustering is a probabilistic approach.

R package MixAll and SaaS MixtComp

Two softwares available

- R package MixAll

```
> library(MixAll)
> data(geyser)
> ## add 10 missing values as random
> x = as.matrix(geyser); n <- nrow(x); p <- ncol(x);
> indexes <- matrix(c(round(runif(5,1,n)), round(runif(5,1,p))), ncol=2);
> x[indexes] <- NA;
> ## estimate model
> model<-clusterDiagGaussian( data=x, nbCluster=2:3, models=c( "gaussian_pk_sjk"))
> plot(model)
> missingValues(model)
  row col     value
1 133   1  2.029661
2  42   2 54.569144
3  49   2 79.970973
4 209   2 54.569144
5 213   2 54.569144
```

- SaaS software MixtComp https://massiccc.lille.inria.fr/

Hypothesis of mixture of parametric distributions

- Cluster k is modeled by a parametric distribution

$$\mathbf{x}_i | z = k \sim p(.|\alpha_k)$$

- Cluster k has probability π_k

$$z_i \sim \mathcal{M}(1, \pi_1, \ldots, \pi_K).$$

Mixture model

The model parameters are $\theta = (\pi_1, \ldots, \pi_K, \alpha_1, \ldots, \alpha_K)$ and

$$p(\mathbf{x}_i) = \sum_{k=1}^{K} \pi_k p(\mathbf{x}_i; \alpha_k)$$

EM Algorithm

Starting from an initial arbitrary parameter θ^0, the mth iteration of the EM algorithm consists of repeating the following **I**, **E** and **M** steps.

- **I step:** *Impute* by using expectation of the missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
- **E step:** Compute conditional probabilities $z_i = k|\mathbf{x}_i$ using current value θ^{r-1} of the parameter:

$$t_{ik}^r = t_k^r(\mathbf{x}_i|\theta^{r-1}) = \frac{p_k^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}{\sum_{l=1}^K p_l^{r-1} h(\mathbf{x}_i|\alpha_k^{r-1})}. \quad (1)$$

- **M step:** Update ML estimate θ^r using conditional probabilities t_{ik}^r as mixing weights

$$L(\theta|\mathbf{x}_1, \ldots, \mathbf{x}_n, \mathbf{t}^r) = \sum_{i=1}^n \sum_{k=1}^K t_{ik}^r \ln\left[p_k h(\mathbf{x}_i|\alpha_k)\right],$$

- Iterate until convergence

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SEM/SemiSEM Algorithms

Drawbacks

- The **I** step may be difficult
- EM algorithm may converges slowly and is slowed down by the imputation step
- Biased estimators

Solution: Use Monte Carlo

- Replace **I** step by a simulation step
- **IS step:** simulate missing values $\mathbf{x}^m$ using $\mathbf{x}^o$, θ^{r-1}, t_{ik}^{r-1}.
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- **S step:** generate labels $\mathbf{z}^r = \{\mathbf{z}_1^r, ..., \mathbf{z}_n^r\}$ according to the categorical distribution $(t_{ik}^r, 1 \leq k \leq K)$.

SEM and SemiSEM does not converge point wise. It generates a Markov chain.

- $\bar{\theta} = (\theta^r)_{r=1,...,R}$
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Mixed Data

Mixed data are handled using conditional independence of the variables.

1. Observation space of the form $\boldsymbol{X} = \boldsymbol{X}_1 \times \boldsymbol{X}_2 \times \ldots \times \boldsymbol{X}_L$
2. $\mathbf{x}_i$ arises from a mixture probability distribution with density

$$f(\mathbf{x}_i = (\mathbf{x}_{1i}, \mathbf{x}_{2i}, \ldots \mathbf{x}_{Li})|\theta) = \sum_{k=1}^{K} \pi_k \prod_{l=1}^{L} h^l(\mathbf{x}_{li}|\boldsymbol{\alpha}_{lk}).$$

3. The density functions (or probability distribution functions) $h^l(.|\boldsymbol{\alpha}_{lk})$ can be any implemented model.

MixAll implements Gaussian, Poisson, Categorical, Gamma distributions.
MixtComp implements Gaussian, Poisson, Categorical and specific distributions for rank and ordinal data.

Sommaire

- Défi Mastodons: Appel à Projet 2016 "Qualité des données"
- Creation of the CloHe (CLustering Of Heterogeneous Data with applications to satellite data records) project
- Members: Mathieu Fauvel (INRA), Stéphane Girard (Inria Grenoble), Vincent vandewalle (Lille2), Crisitan Preda (Université Lille 1)

`https://modal.lille.inria.fr/CloHe/`

Formosat-2 is no more operational

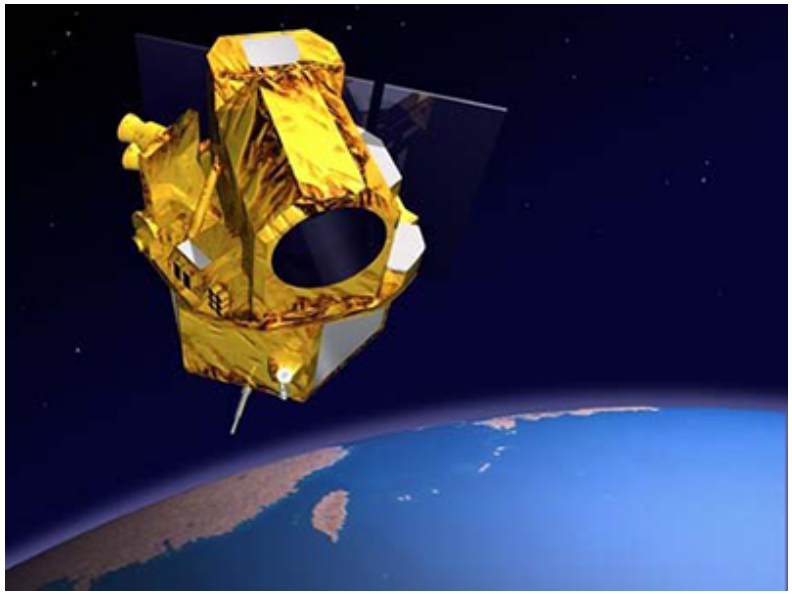


Figure: Formosat-2 furnished multi-spectral data (R, G, B, NIR) with a 8 meter resolution. 17 complete images of France by year

Sentinel-2A start service in 2016.

Figure: Sentinel-2 furnish 13 spectral bandwidths with 4 bandwidths with a 10 meters resolution and 6 bandwidths with a 20 meters resolution. A complete image of France every 5 days

Data Cube

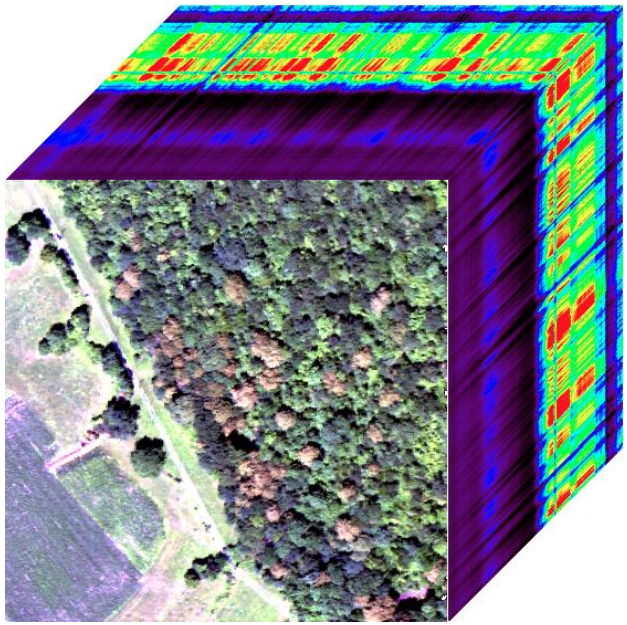


Figure: Sentinel-2 furnish approximately 20TB of images/year, and cover the entire France in 5 days with 1.6 milliard de pixels.

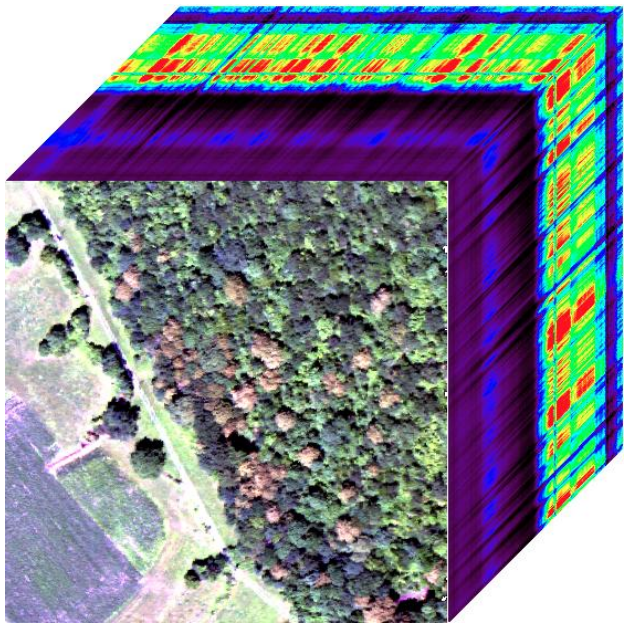
Data Cube

$$\mathbf{X} = (X_{ikt}), \quad i \in I, \quad k \in \{\text{r,v,b,ir}\},$$
$$\mathbf{Y} = (Y_i), \quad i \in J \subset I.$$

with

- $i = (x, y)$ geographic position,
- k spectral band,
- t dates,
- missing values (clouds, ported shadows) at some dates and some positions,
- noisy data (undetected shadows, cloud veil, etc...),
- mixel (mixture of pixel)

Data Cube



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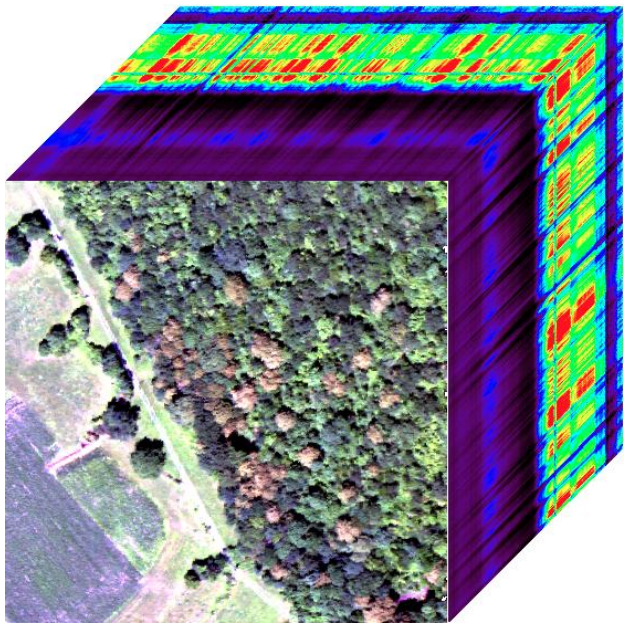

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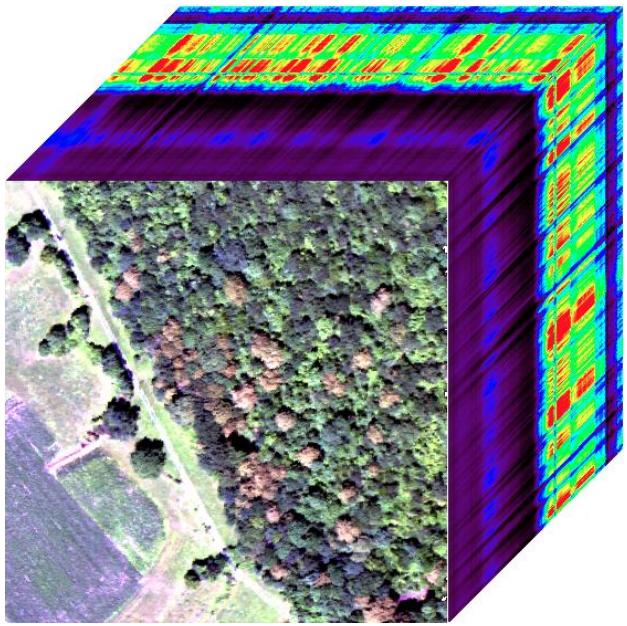

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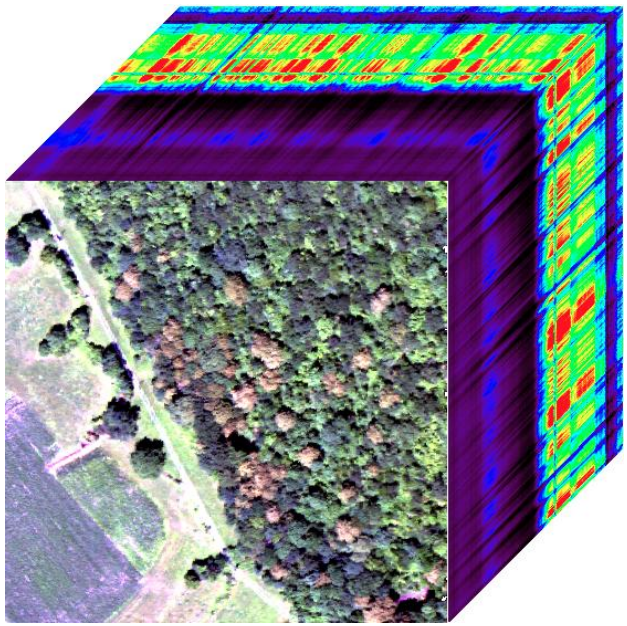

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Missing data

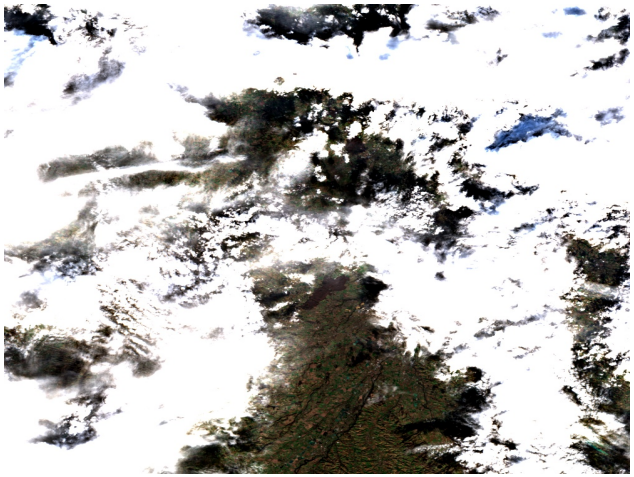


Figure: Very cloudy

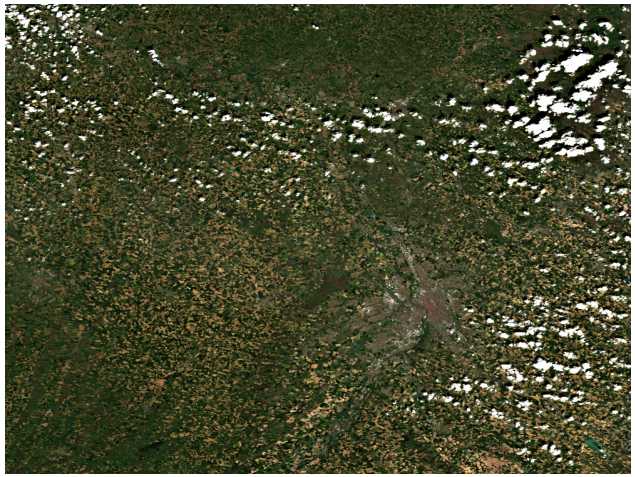


Figure: A few number of clouds

Figure: "sheeps"

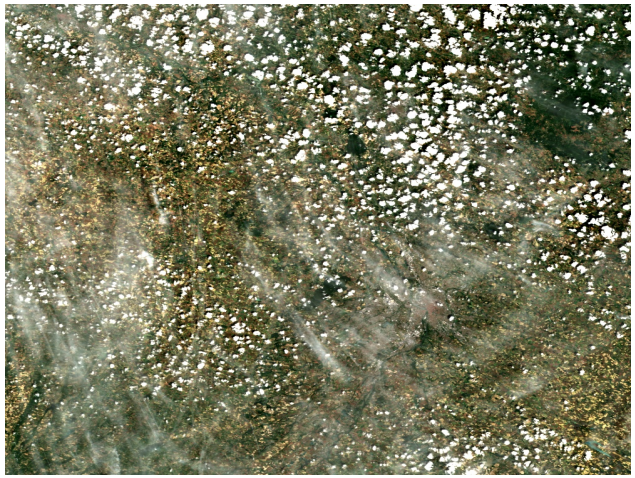


Figure: Some clouds with a veil

Noisy Data

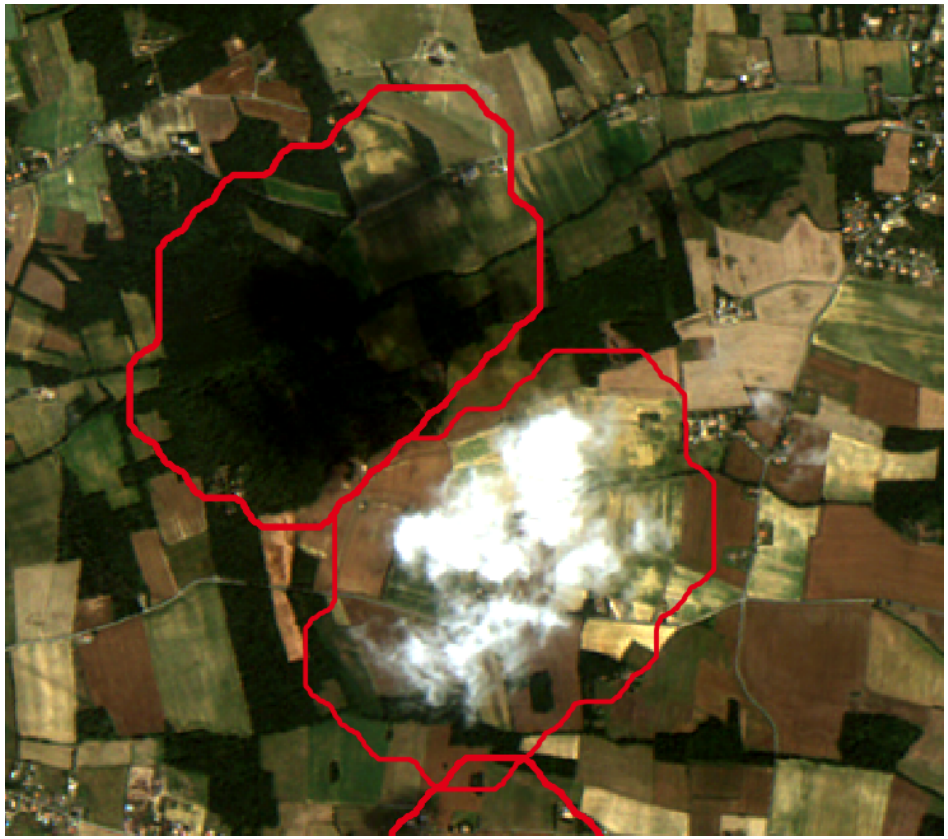


Figure: clouds and their shadows

Non-Uniform sampling

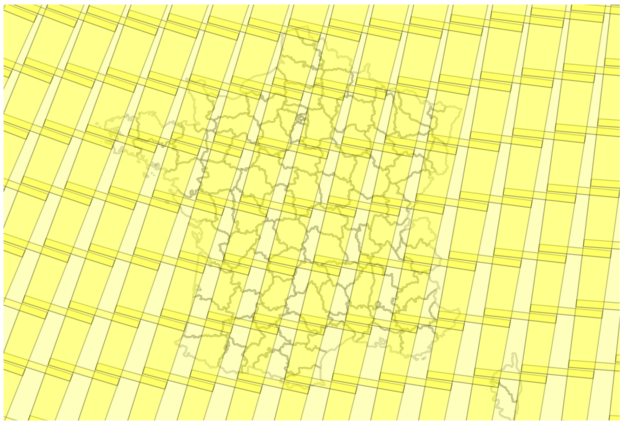


Figure: Path-row grid for Landsat acquisitions. Every path (North-South track) is acquired on the same date every 16 days.

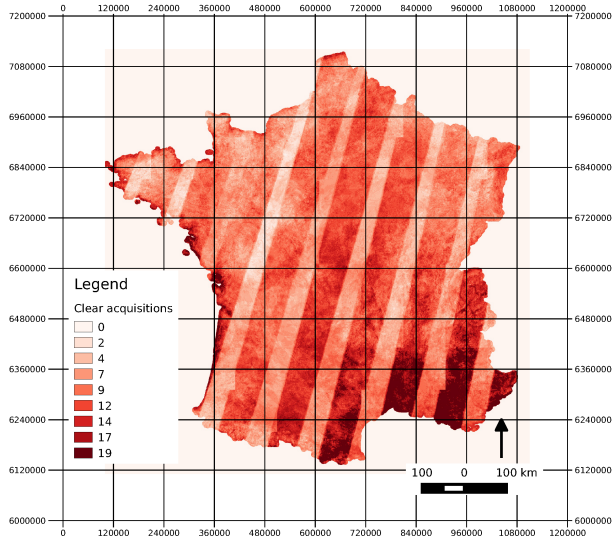


Figure: Map of the number of times that every pixel sees the ground taking into account satellite revisit and cloud cover.

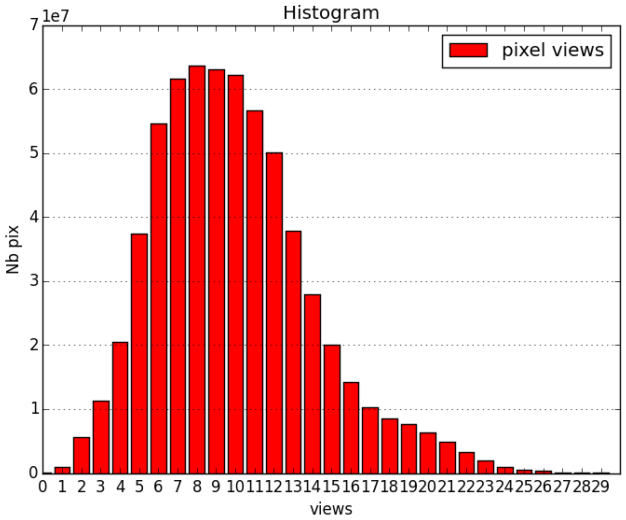
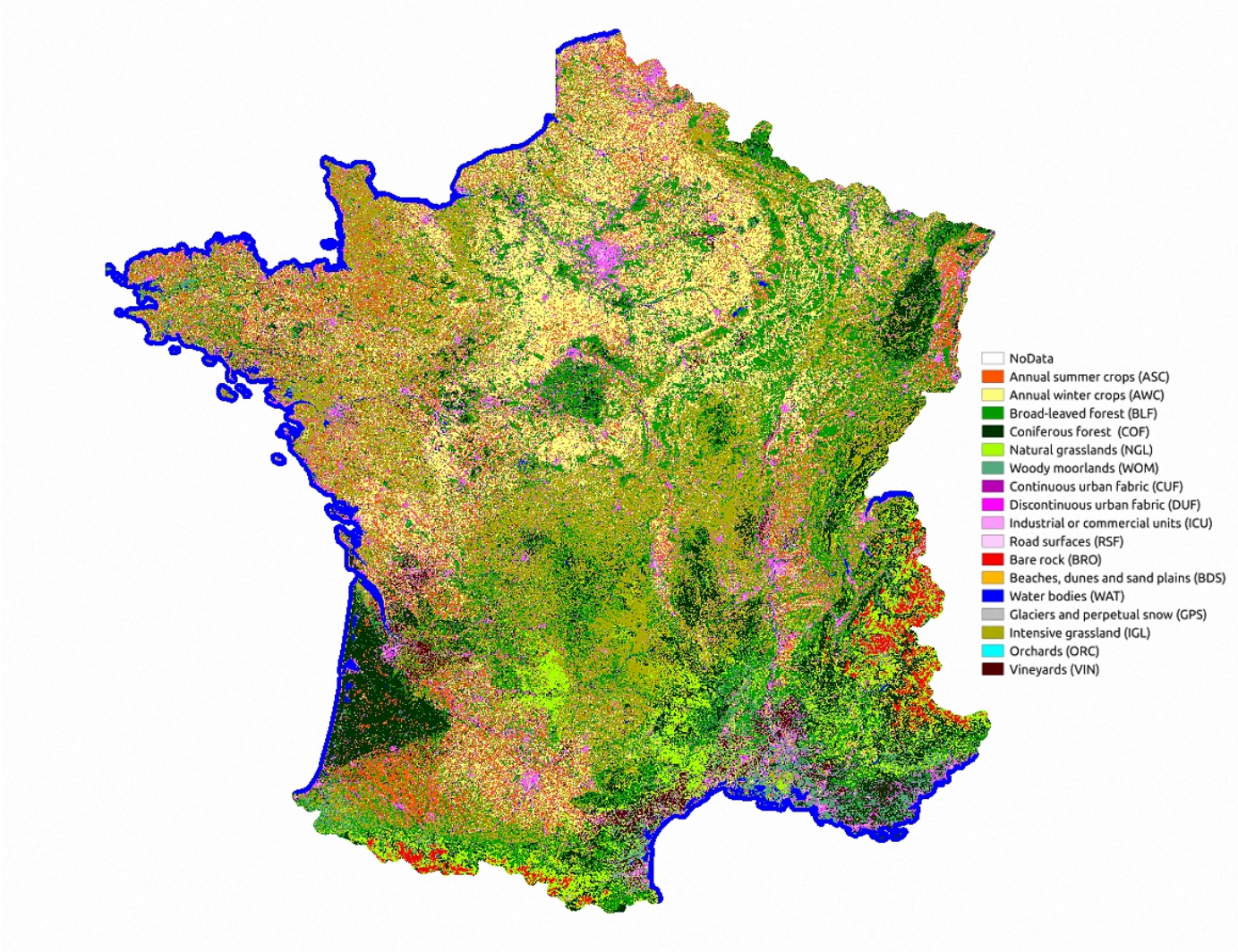


Figure: Histogram of the number of times that every pixel sees the ground taking into account satellite revisit and cloud cover.

Open Access: `http://www.mdpi.com/2072-4292/9/1/95/htm`

Objective

The aim is to be able to cluster the whole France using Sentinel-2 data.



Gaussian modeling

- $Y_i \in \{1, \ldots, G\}$,
- $\mathcal{L}(\mathbf{X}_i | Y_i = g) = \mathcal{N}(\boldsymbol{\mu}_g, \Sigma_g)$
- Two kinds of parsimony assumptions on covariance matrices
  - independence between spectra $\Sigma_{g,k}$ of size $T \times T$, $(T = 17)$,
  - or independences between times, $\Sigma_{g,t}$ of size $K \times K$, $(K = 4)$.
- handle missing values for both models
- Implementations and tests in a R package.

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Method

Missing values formation process

Missing At Random (MAR): Probability for a value to be missing does not depends from its value conditionally to the other observations.

Denote $\boldsymbol{x}_{ik}^{+} = \begin{pmatrix} \boldsymbol{x}_i^{\mathrm{O}} \\ \boldsymbol{x}_{ik}^{\mathrm{M}^+} \end{pmatrix}$, $\tilde{\boldsymbol{\Sigma}}_{ik}^{+} = \begin{pmatrix} 0_i^{\mathrm{O}} & 0_i^{\mathrm{OM}} \\ 0_i^{\mathrm{MO}} & \tilde{\boldsymbol{\Sigma}}_{ik}^{\mathrm{M}^+} \end{pmatrix}$ with 0 null matrix, and $\tilde{\boldsymbol{\Sigma}}_{ik}^{\mathrm{M}^+} = \boldsymbol{\Sigma}_{ik}^{\mathrm{M}} - \boldsymbol{\Sigma}_{ik}^{\mathrm{MO}} \left(\boldsymbol{\Sigma}_{ik}^{\mathrm{O}}\right)^{-1} \boldsymbol{\Sigma}_{ik}^{\mathrm{OM}}$. then

$$\boldsymbol{\Sigma}_k^{+} = \frac{1}{n_k^{+}} \sum_{i=1}^{n} \left[(\boldsymbol{x}_{ik}^{+} - \boldsymbol{\mu}_k^{+})(\boldsymbol{x}_{ik}^{+} - \boldsymbol{\mu}_k^{+})' + \tilde{\boldsymbol{\Sigma}}_{ik}^{+} \right]$$

$\tilde{\boldsymbol{\Sigma}}_{ik}^{\mathrm{M}^+}$

is correcting the variance due to the imputation by the mean.

Gaussian modeling

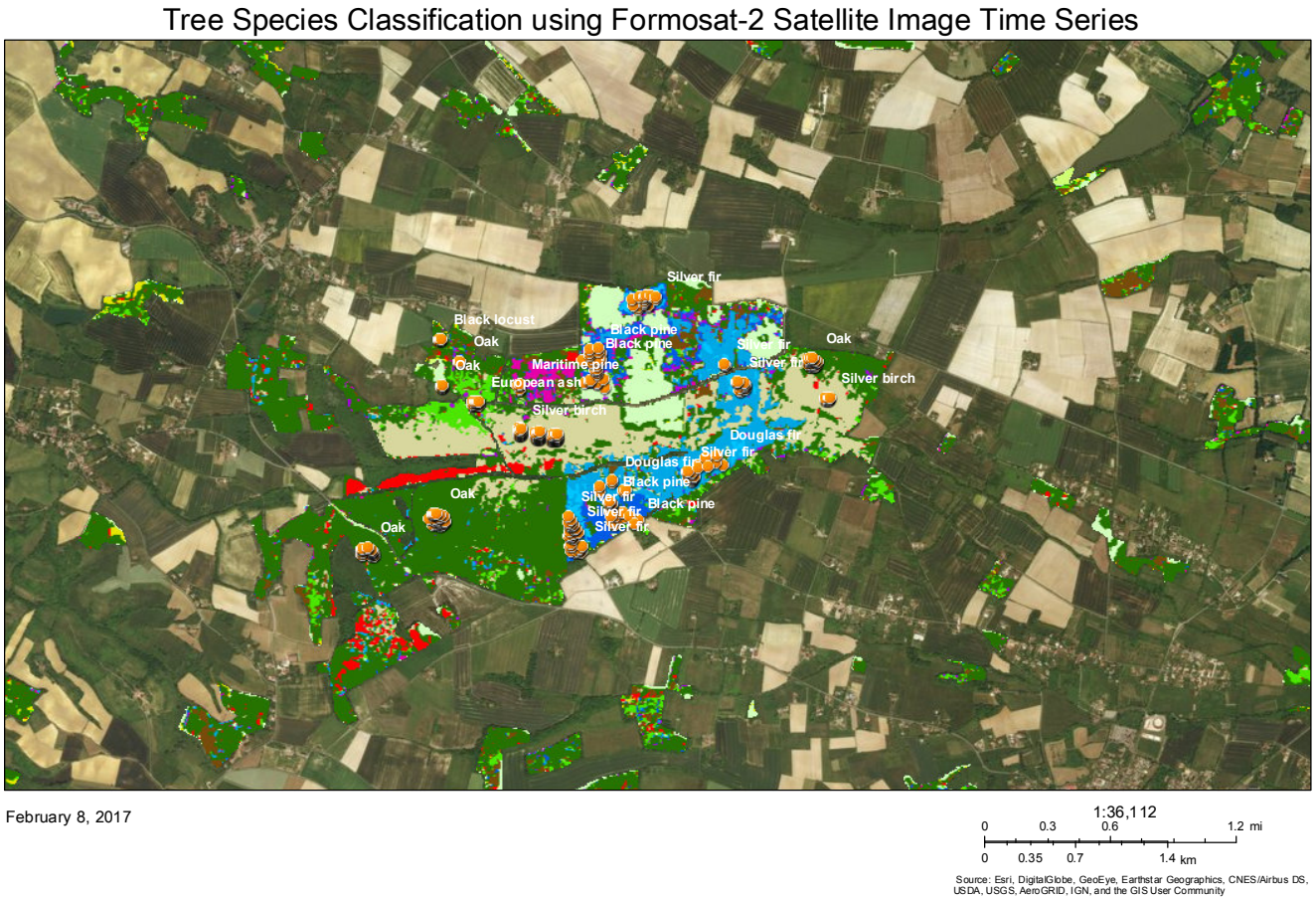

Figure: Tree species classification with $G = 13$

Continuous Model

Main assumption

$$Y|Z = k \sim GP(\mu_k, C_k),\ k = 1, \ldots, K \tag{2}$$

where $GP(\mu_k, C_k)$ is a Gaussian Process with mean $\mu_k \in L_2(I)$ and with covariance operator $C_k : I \times I \to \mathbb{R}$.

- mean functions belongs to a $J-$dimensional subspace

$$\mu_k(t) = \sum_{j=1}^{J} \alpha_{kj}\varphi_j(t),$$

- Covariance function

$$C_k(s,t)(h_k) = \theta_k Q((t-s)/h_k),$$

- Spectrum are independents.

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Estimation of Continuous Model

For each i, let $B^i_{\ell,j} = \varphi_j(t^i_\ell)$, $m_{ki} = B^i \alpha_k$ and

$$\Sigma^i_{j,j'}(h_k) = \theta_k Q((t^i_j - t^i_{j'})/h_k) =: \theta_k S^i_{j,j'}(h_k),$$

then

$$y_i | Z_i = k \sim \mathcal{N}_{T_i}(m_{ki}, \theta_k S^i(h_k)), \ k = 1, \ldots, K, \ i = 1, \ldots, n$$

we end up with K independent minimization problems:

$$(\hat{\alpha}_k, \hat{h}_k) = \arg \max_{\alpha_k, h_k, \theta_k} \sum_{Z_i = k} \log\det S^i(h_k) + T_i \log \theta_k + \frac{1}{\theta_k}(y_i - B^i\alpha_k)^\top S^i(h_k)^{-1}(y_i - B^i\alpha_k)$$

Results

About 65% well classified.

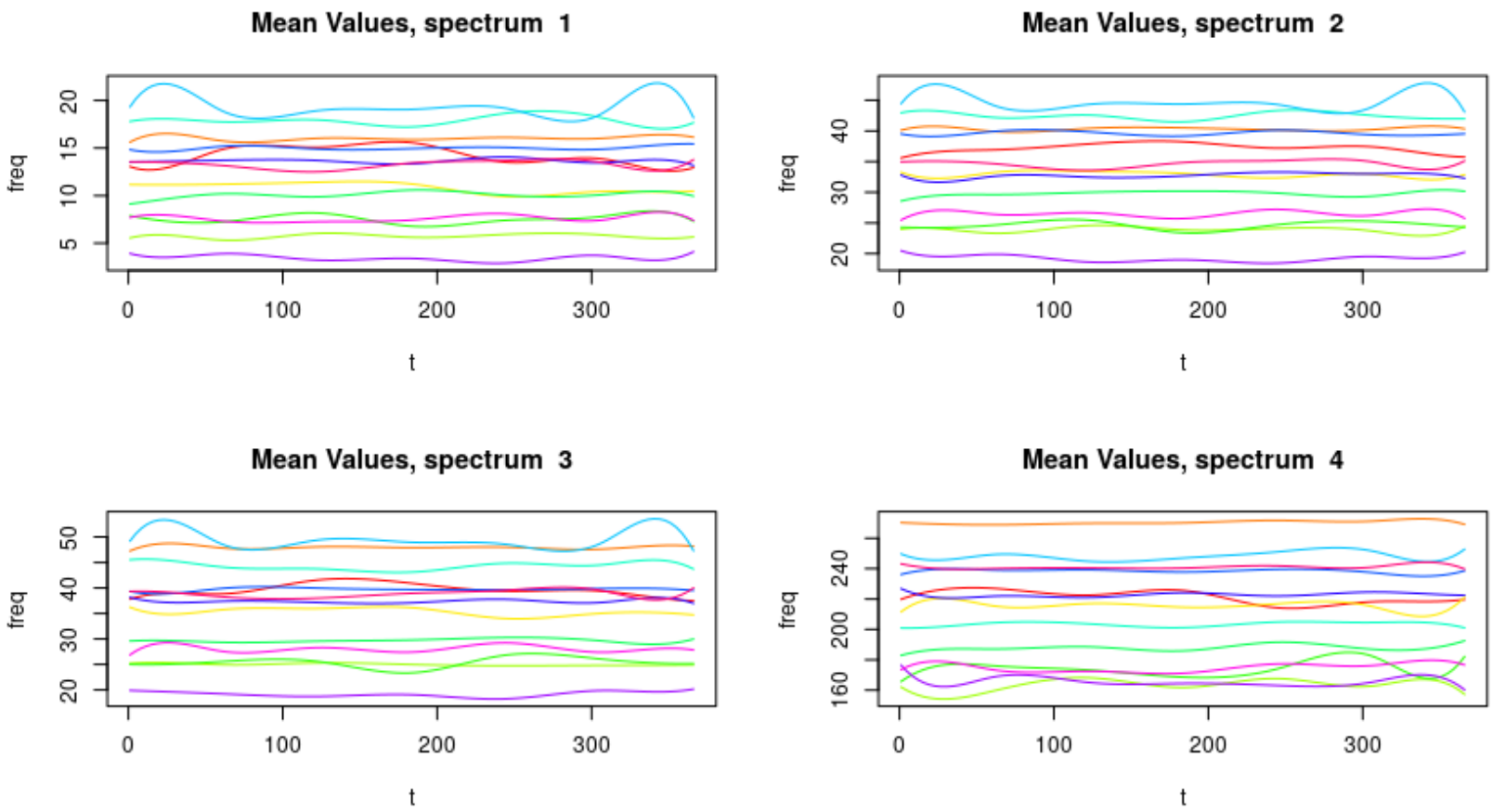

Figure: $G = 13$ spectrum

Mean values

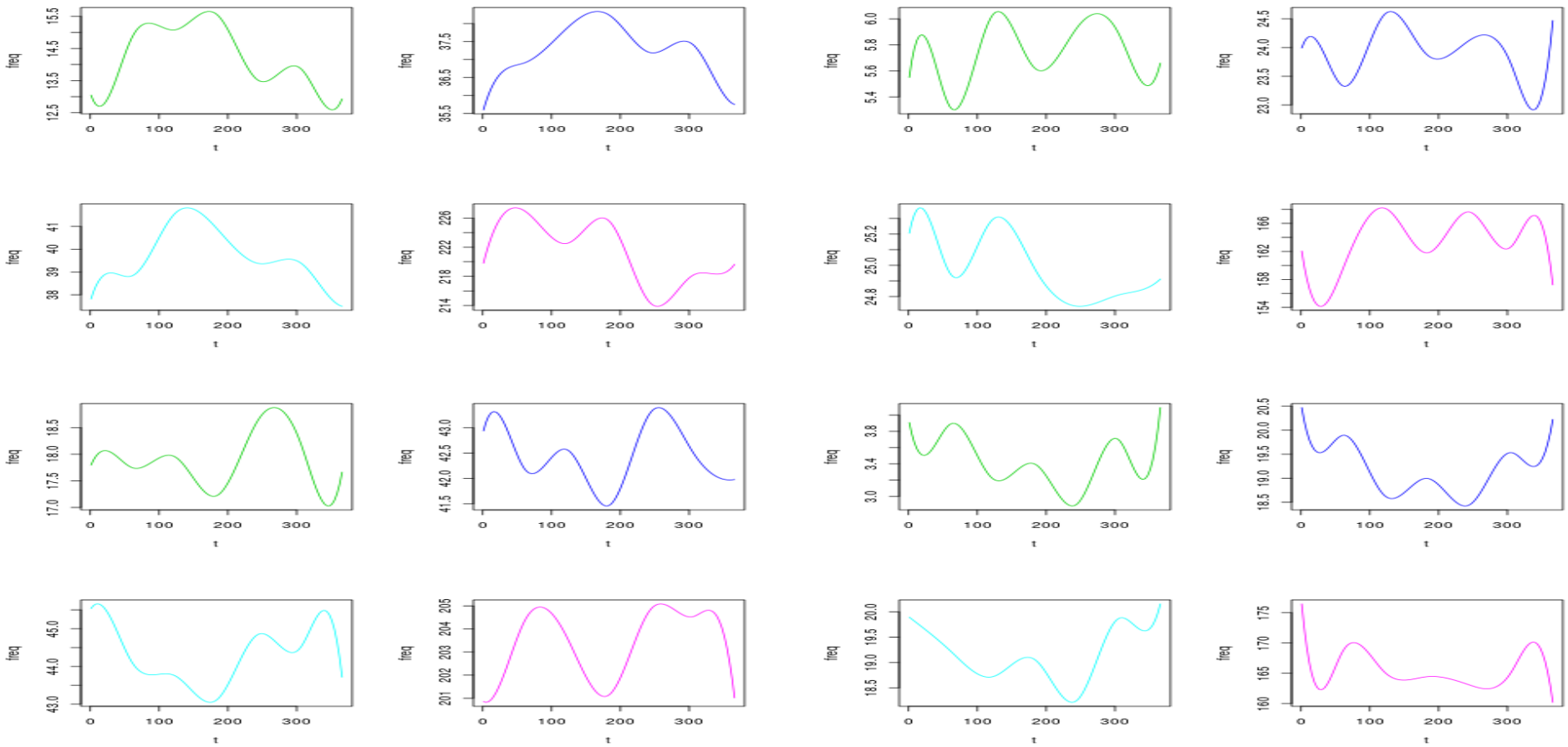

Figure: first, 4th, 7th and 11th classes

Links

- https://cran.r-project.org/web/packages/MixAll/
- https://massiccc.lille.inria.fr/
- https://modal.lille.inria.fr/CloHe/
- http://www.mdpi.com/2072-4292/9/1/95/htm